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APPLICATION OF CONFORMAL MAPPING AND VARIATIONAL METHOD TO THE STUDY OF HEAT CONDUCTION IN POLYGONAL PLATES WITH TEMPERATURE/DEPENDENT CONDUCTIVITY

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Abstract—The stationary value problem of the local potential for heat conduction in a polygonal plate with temperature-dependent conductivity is transformed by a holomorphic function into an equivalent stationary value problem for another plate with a circular boundary. The equivalent problem is then solved by the Rayleigh–Ritz method since the coordinate functions in the circular region are readily available. This method offers a unified approach to the problem of the temperature distribution of one plate and of all others. The application of this method can be easily extended to other transport phenomena which are governed by the extremization of a functional.

NOMENCLATURE

- a_c, characteristic dimensions of a polygonal plate;
- $f'(\zeta)$, derivative of any function $f(\zeta)$ with respect to the variable ζ;
- $F(\alpha_i)$, function of the undetermined coefficients α_i;
- $F[\theta]$, functional of the function θ;
- $J_m(r)$, Bessel function of order m;
- k, κ, thermal conductivity and diffusivity;
- k_0, κ_0, reference values of k and κ;
- $\bar{Q}$, complex conjugate of any quantity Q;
- $|Q| = (Q\bar{Q})^{\frac{1}{2}}$, modulus of any quantity Q;
- $Q_{(jk)}$ symmetric part of any quantity Q_{jk};
- r, ψ, polar coordinates in the ζ-plane;
- t, τ, time and dimensionless time;
- T, T^*, temperatures subject to and not subject to variations;
- T_s, specified initial temperature;
- u, v, dimensionless coordinates in a rectangular system in the z-plane;
- x, y, rectangular coordinates;
- z, ζ, complex variables;
- $z = f(\zeta)$, conformal mapping function;
- θ, θ^*, dimensionless temperatures subject to and not subject to variations;
- ξ, η, coordinates in a rectangular system in the ζ-plane;
- ρ, c_v, constant density and specific heat;
- $\phi^n(\zeta, \bar{\zeta})$, coordinate functions.

1. INTRODUCTION

THE LINEAR problem of heat conduction in a plate which has a boundary that coincides with the coordinate curves has been studied extensively from both the engineering and the mathematical viewpoints [1, 2]. Analytic solutions are not tenable for polygonal plates. Laura and Faulstich [3] present solutions for this important class of plates with constant conductivity. These authors apply the Munakata approach [4] to transform the differential equation by a holomorphic function, which maps the region of the definition of the original equation into a circular region; the solution of the transformed equation is then approximated by the Galerkin method. A fairly complete bibliography on the application of conformal mapping in this direction can be found in [5–7]. This paper presents another aspect of the application of conformal mapping. In this new approach, a functional which is defined in an irregular plane region and requires a stationary value, is conformally mapped onto a circular region. The functional which is now defined in a circular region, is extremized by the Rayleigh–Ritz method since the coordinate functions in a circular region can be easily chosen. This method is successfully applied to the determination of the buckling loads of polygonal plates in [8].

Up until the local potential was introduced by Glansdorff *et al.* [9], applications of the variational method were largely restricted to the area of solid mechanics. Hays and Curd have applied this extended variational method to heat conduction in solids [10], to diffusional problems [11, 12], and to hydrodynamics [13]. Thus far, the extended variational method has been applied to problems with relatively simple geometric boundaries; that is, either square, circular or semi-infinite. It is the primary purpose of this paper to apply the conformal transformation to the local potential so that problems in a plane region with an irregular boundary can be investigated. As a specific illustration, the problem of unsteady heat conduction in polygonal plates with temperature-dependent conductivity is studied. Other transport problems can be formulated with a similar procedure.

2. GENERAL FORMULATION

Let an elastic plate in a temperature field occupy a region R with an irregular boundary C, as shown in Fig. 1a. It has been shown by Hays [14] that the macroscopic temperature distribution in R with the temperature of flux specified on C can be obtained by extremizing the following functional:

$$F[T] = \int_0^\infty \iint_R \left\{ \tfrac{1}{2}k(T^*)\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \rho c_v T \frac{\partial T^*}{\partial t}\right\} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \tag{1}$$

subject to the initial condition and the subsidiary condition $T^* = T$ after the variation process.

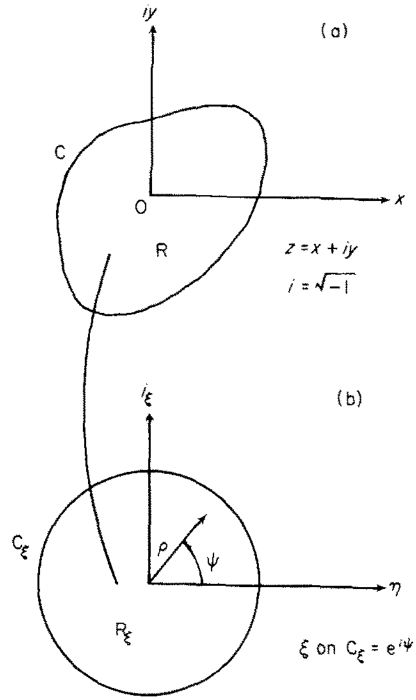

FIG. 1. Conformal transformation of region R onto a unit circular region R_ξ.
(a) Region of actual plate.
(b) Region of transformed plate.

The thermal diffusivity will be assumed to be a linear function of temperature which takes the form

$$\kappa = \frac{k}{\rho c_v} = \kappa_0(1 + \sigma\theta) \tag{2}$$

where σ is a free parameter characterizing the slope of the diffusivity–temperature curve.

It is expedient to introduce the following dimensionless variables:

$$u = \frac{x}{a_c} \tag{3a}$$

$$v = \frac{y}{a_c} \tag{3b}$$

$$\tau = \frac{\kappa_0}{(a_c)^2}t \tag{3c}$$

and

$$\theta = \frac{T}{T_s}. \tag{3d}$$

Substitution of equation (2) and (3) into equation (1) yields

$$F[\theta] \quad \int_0^\infty \iint_R \left\{\tfrac{1}{2}(1+\sigma\theta^*)\left[\left(\frac{\partial\theta}{\partial u}\right)^2 + \left(\frac{\partial\theta}{\partial v}\right)^2\right] + \theta\frac{\partial\theta^*}{\partial\tau}\right\} du\, dv\, d\tau. \tag{4}$$

The temperature T must be equal to T_s in R + C at $t = 0$ and equal to zero on C for $t > 0$. Consequently, the dimensionless temperature must satisfy the initial and the boundary conditions as follows:

$$\theta = \theta^* = 1 \quad \text{in R at } t = 0 \tag{5a}$$

$$\theta = \theta^* = 0 \quad \text{on C for } t > 0. \tag{5b}$$

If the coordinate functions which satisfy the completeness condition in R and the conditions in equation (5b) are chosen, then the functional in equation (4) can be extremized by the Rayleigh–Ritz method. The choice of the coordinate functions for a simple shape of R is demonstrated in [10, 13]. The coordinate functions for a complicated shape can be formulated according to the guides proposed by Yoshiko and Kawai [15]. However, the method in [15] is undesirable since the chosen functions must be formulated once for each plate and the complicated manipulation for the extremizing process must be executed for each problem. In this paper, a method is presented which carries out the complicated extremizing process for one plate and for all other plates with boundaries which form the conformal images of a unit circle.

If the region R can be obtained by transforming a unit circular region R_ζ, as shown in Fig. 1b, by a holomorphic function

$$w = \frac{z}{a_c} = \frac{1}{a_c}f(\zeta) \quad \text{with } f'(\zeta) \neq 0 \text{ in } R_\zeta \tag{6}$$

then equation (4), after some manipulations, reduces to

$$F[\theta] = \int_0^\infty \int_0^{2\pi} \int_0^1 \left\{2(1+\sigma\theta^*)\left|\frac{\partial\theta}{\partial\zeta}\right|^2 + \frac{1}{(a_c)^2}|f'(\zeta)|^2\theta\frac{\partial\theta^*}{\partial\tau}\right\} r\, dr\, d\psi\, d\tau \tag{7}$$

where $\zeta = r\,e^{i\psi}$. The functional (7) is now defined in a circular region for which the coordinate functions can be easily chosen.

The temperature distribution of a circular plate with the boundary conditions (5b) can be expressed as a linear combination of the zero order Bessel functions [1]. Consequently, by neglecting the variation of θ with respect to the polar angle ψ, the coordinate functions for the functional (7) in a circular region can be expressed as

$$\theta = \sum_{m=1}^\infty A_m J_0(c_m r)\exp(-\alpha_m \tau) \tag{8a}$$

$$\theta^* = \sum_{m=1}^\infty A_m J_0(c_m r)\exp(-\beta_m \tau) \tag{8b}$$

where c_m is one of the roots of $J_0(c_m) = 0$ which are in the order of $c_1 < c_2 < \ldots < c_m < \ldots$. To satisfy the subsidiary condition, β_m will be set equal to α_m after the variational process. Substitution of equation (8a) into equation (5a) for satisfaction of the initial condition yields

$$A_m = \frac{2}{c_m J_1(c_m)}. \tag{9}$$

To provide a specific illustration, a class of regular polygonal plates will be studied. The mapping function which conformally transforms a regular polygonal region onto a unit circle is

$$f(\zeta) = a_c \Gamma \int_0^{\zeta} \frac{\mathrm{d}\varepsilon}{(1 - \varepsilon^{\Delta})^{2/\Delta}} \qquad (10)$$

where Δ is the number of sides, a_c is the apothem, and Γ represents the mapping coefficients [7] in Table 1.

Table 1. Mapping coefficients Γ for regular polygons

Shape	+
Triangle	1·135
Square	1·079
Pentagon	1·052
Hexagon	1·038
Heptagon	1·028
Octagon	1·022

If the binomial formula is used, the derivative of the mapping function (10) can be expressed as follows:

$$f'(\zeta) = a_c \Gamma \sum_{n=0}^{\infty} b_n (\zeta^{\Delta})^n \qquad (11)$$

where

$$b_0 = 1$$

and

$$b_n = \frac{2}{n!\Delta^n}(2 + \Delta)(2 + 2\Delta)\ldots[2 + (n - 1)\Delta].$$

It can be seen from equation (7) that only the derivative of the mapping function is needed. Equation (11) is specifically designed for regular polygons; however, it can be used for other regions by simply taking $a_c = \Gamma = \Delta = 1$ and assigning the proper values to the coefficients b_n.

By substitution of equations (8) and (11) into equation (7), the functional $F[\theta]$ becomes a function of α_i denoted by $F(\alpha_i)$. Consequently, the conditions for a stationary value of $F[\theta]$ reduce to

$$\frac{\partial F(\alpha_i)}{\partial \alpha_j} = 0 \qquad j = 1, 2, \ldots, N \qquad (12)$$

for an N-term approximation. If the detailed operations in equation (12) are performed and β_i is then set equal to α_i, the following equations for α_i result:

$$F_j(\alpha_1, \alpha_2, \ldots, \alpha_N; \sigma) = \sum_{k=1}^{N} \frac{A_k B(jk)}{(\alpha_j + \alpha_k)^2} + \sum_{i,k=1}^{N} \frac{\sigma A_i A_k C_i(jk)}{(\alpha_i + \alpha_j + \alpha_k)^2} - \frac{\Gamma^2}{4} \sum_{k=1}^{N} \sum_{p,q=0}^{\infty} \frac{A_k b_p b_q D(jk)(pq)\,\alpha_k}{(\alpha_j + \alpha_k)^2} = 0 \qquad j = 1, 2, \ldots N \qquad (13)$$

where

$$B_{jk} = \int_0^{2\pi} \int_0^1 \frac{\partial P_j}{\partial \zeta} \left(\overline{\frac{\partial P_k}{\partial \zeta}}\right) r\,\mathrm{d}r\,\mathrm{d}\psi \qquad (14a)$$

$$C_{ijk} = \int_0^{2\pi} \int_0^1 P_i \frac{\partial P_j}{\partial \zeta} \left(\overline{\frac{\partial P_k}{\partial \zeta}}\right) r\,\mathrm{d}r\,\mathrm{d}\psi \qquad (14b)$$

$$D_{jkpq} = \int_0^{2\pi} \int_0^1 P_j P_k (\zeta^{\Delta})^p \overline{(\zeta^{\Delta})^q}\, r\,\mathrm{d}r\,\mathrm{d}\psi \qquad (14c)$$

$$P_j = J_0(c_j r). \qquad (14d)$$

If one multiplies both sides of equation (13) by $(\alpha_j)^2$ and observes the relations

$$B_{jk} = 0 \quad \text{if} \quad j \neq k \qquad (15a)$$

$$B_{jj} = \frac{\pi}{4}[c_j J_1(c_j)]^2 \qquad (15b)$$

$$D_{jj00} = \pi[J_1(c_j)]^2. \qquad (15c)$$

Equation (13) can be rewritten as follows:

$$F_j(\alpha_1, \alpha_2, \dots, \alpha_N; \sigma) = \frac{\pi A_j}{16}[J_1(c_j)]^2 [(c_j)^2 - \Gamma^2\alpha_j] - \frac{\Gamma^2}{4}\sum_{k=1}^{N}\sum_{p,q=1}^{\infty} A_k b_p b_q D_{(jk)(pq)} \frac{\alpha_k(\alpha_j)^2}{(\alpha_j + \alpha_k)^2} + \sigma \sum_{i,k=1}^{N} A_i A_k C_{i(jk)} \frac{(\alpha_j)^2}{(\alpha_i + \alpha_j + \alpha_k)^2} = 0 \qquad j = 1, 2, \dots, N. \tag{16}$$

Equation (16) is a set of N nonlinear algebraic equations for N unknowns α_j, with σ and $b_1, b_2, \dots, b_\infty$ as small parameters. Consequently, the solution of equation (16), in the general case, can be generated from those cases where σ and $b_1, \dots, b_\infty$ are zero. By setting those small parameters equal to zero and setting Γ equal to one in equation (16), one obtains $\alpha_j = (c_j)^2$ which is the expected solution of α_j for a circular plate with constant conductivity.

3. NUMERICAL RESULTS

The procedure for solution of equation (16) with the arbitrary value of σ will be developed according to the Newton–Raphson method [10, 14]. If the solution of equation (16) with $\sigma = \sigma_0$ has been obtained and is denoted by $\alpha_i^{(0)}$, then

$$F_j(\alpha_1^{(0)}, \dots, \alpha_N^{(0)}; \sigma_0) = 0. \tag{17}$$

And, if the unknown solution for σ which is very close to σ_0 is α_i, α_i must satisfy the following equations:

$$F_j(\alpha_1, \dots, \alpha_N; \sigma) = 0. \tag{18}$$

The assumption that the α_i are in the neighborhood of $\alpha_i^{(0)}$ and the expansion of equation (18) in a Taylor series with only the terms of the first order smallness result in

$$F_j(\alpha_1, \dots, \alpha_N; \sigma) = F_j(\alpha_1^{(0)}, \dots, \alpha_N^{(0)}; \sigma_0) + \frac{\partial F_j}{\partial \sigma_0}\Delta\sigma + \sum_{k=1}^{N} \frac{\partial F_j}{\partial \alpha_k^{(0)}}\Delta\alpha_k = 0 \tag{19}$$

where

$$\Delta\sigma_k = \alpha_k - \alpha_k^{(0)} \tag{20a}$$

$$\Delta\sigma = \sigma - \sigma_0. \tag{20b}$$

The step size of $\Delta\sigma$ should be small enough so that equation (19) is valid within a certain preassigned accuracy. With the use of equation (17), the increments $\Delta\alpha_k$ can be solved from equation (19) by the Cramer rule,

$$\Delta\alpha_k = \sum_{k=1}^{N} I_{kj}\frac{\partial F_j}{\partial \sigma_0} \quad j = 1, 2, \dots, N \tag{21}$$

where I_{jk} are the elements of the inverse of the matrix $-\partial F_j/\partial \alpha_k^{(0)}$. After the $\Delta\alpha_k$ are determined, the values of α_k for an arbitrary σ can thus be obtained from equation (20a).

Since, from the beginning, the values of $\alpha_i^{(0)}$ are assumed to be known, the problem becomes one of determining the $\alpha_i^{(0)}$. They can always be obtained through a similar procedure with $\sigma_0 = 0$ by expanding F_j into a Taylor series with $b_1, b_2, \dots, b_M$ as the parameters. This choice of parameters is always possible since $b_M \leqslant b_{M-1} \leqslant \dots \leqslant b_2 \leqslant b_1 < 1$.

For the evaluations of $\Delta\alpha_k$, the derivatives of F_j with respect to α_k are needed. Through some rearrangements, they reduce to

$$-\frac{\partial F_j}{\partial \alpha_k} = \frac{2\pi A_j}{16}[\Gamma J_1(c_j)]^2\delta_{jk} + \sum_{i=1}^{N} A_k A_i C_{(kji)} \frac{4\sigma(\alpha_j)^2}{(\alpha_k + \alpha_j + \alpha_i)^2}$$

$$- \sum_{i,m=1}^{N} A_m A_i C_{m(ji)} \frac{2\sigma(\alpha_m + \alpha_i)\,\alpha_j \delta_{jk}}{(\alpha_m + \alpha_j + \alpha_i)^3}$$

$$+ \frac{\Gamma^2}{4} \sum_{p,q=1}^{\infty} A_k b_p b_q D_{(jk)(pq)} \frac{(\alpha_j)^2(\alpha_j - \alpha_k)}{(\alpha_j + \alpha_k)^3}$$

$$+ \frac{\Gamma^2}{4} \sum_{i=1}^{N} \sum_{p,q=1}^{\infty} A_i b_p b_q D_{(ij)(pq)} \frac{2(\alpha_i)^2 \alpha_j \delta_{jk}}{(\alpha_i + \alpha_j)^3}$$

$$j, k = 1, 2, \ldots, N. \qquad (22)$$

To indicate the convergence of the series solution for the temperature distribution, the first ten coefficients α_i of a square plate with different values of σ are listed in Table 2. The rapid increase in the magnitude of α_i shows that the ten term approximation gives a good expression of the temperature distribution. Substitution of the α_i coefficients, thus obtained, back into equation (16) yields ten residues which indicate the degrees of dissatisfaction of the ten nonlinear equations. The arithmetic means of the residues are listed at the bottom of Table 2. It is seen that the mean residues which indicate the degrees of dissatis-negligible when compared with even the smallest coefficient α_1.

The dimensionless temperature distribution of the regular polygonal plates with different values of σ are calculated in the transformed plane by an IBM 360 computer. Some of the results, which illustrate the basic natures of the parameter influences, are summarized in Figs. 2

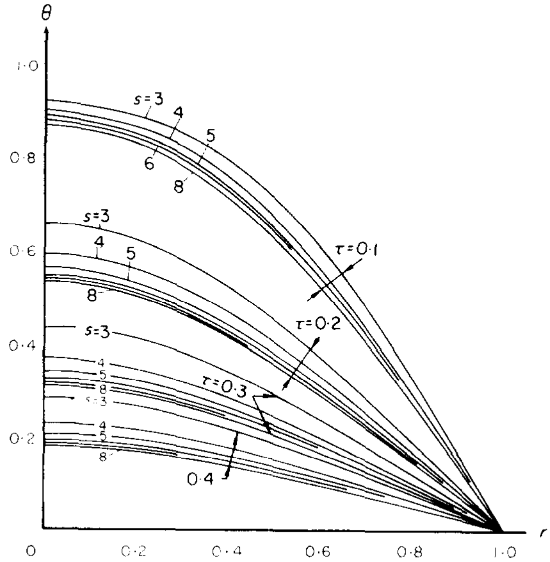

FIG. 2. Temperature distribution of regular polygonal plates with $\sigma = 0$.

and 3. Figure 2 shows that the temperature difference between two neighboring curves at the same position first increases and then decreases as time is increasing. This indicates the expected phenomenon; that is, the effect

Table 2. Coefficients α_i and means of residues for a square plate

α_i	σ					
	0·0	0·2	0·4	0·6	0·8	1·0
α_1	5·0418	5·2846	5·5265	5·7677	6·0082	5·2062
α_2	24·823	24·939	25·150	25·535	25·777	26·911
α_3	57·685	57·439	57·356	57·419	57·613	64·446
α_4	102·24	101·60	101·16	100·92	100·87	116·73
α_5	158·15	157·37	156·79	156·41	156·25	183·28
α_6	225·66	225·32	225·09	225·01	225·12	264·07
α_7	305·93	307·69	309·27	310·79	312·33	359·48
α_8	399·93	405·09	409·69	413·89	417·86	470·29
α_9	513·10	529·04	543·43	556·63	568·92	579·95
α_{10}	634·98	644·01	652·20	659·75	666·85	744·29
Residue $\times 10^8$	$-0·459$	$+1·234$	$+2·461$	$+0·191$	$-6·405$	$-3·814$

of the shape of the plate on the temperature distribution is at first pronounced, then weakens. Figure 3 shows that, at a fixed time and a fixed position in the transformed region R_ζ, the temperature differences among all curves with equal increments of σ may be expected to be the same. However, if the curves are transformed into the actual plane R, the actual temperature differences will no longer be the same but will be magnified by a factor which is a function of the coefficients α_i. The value of this factor can be evaluated from equation (8a).

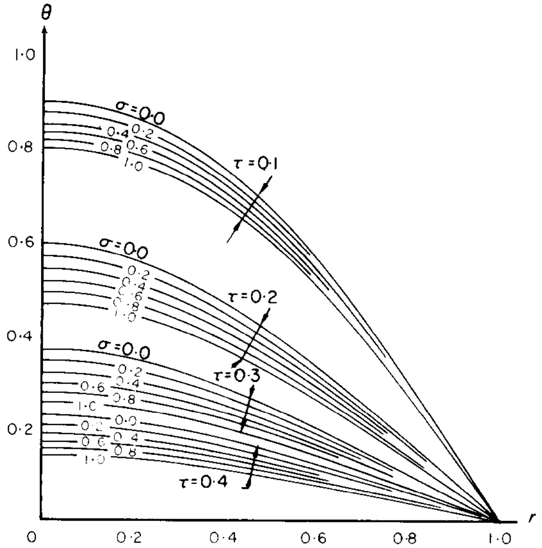

FIG. 3. Temperature distribution of a square plate with different values of α.

4. CONCLUSIONS

The temperature distribution in the polygonal plates with linear temperature-dependent conductivity has been solved once and for all. The development is equally valid for cases in which the conductivity is an arbitrary function of temperature. In the general cases, the algebraic equations are more complicated, but this will introduce no basic difficulty.

Without reiterating the many advantages of the variational method, the procedure developed in this paper provides the following significant features: simplicity of development, unification of the various plate shapes, and finally, economy of time required for a numerical result.

Since the method is simply to determine the stationary value of a functional, it can be applied to any problem in a plane region with an irregular boundary if the functional associated with the problem is known.

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REFERENCES

1. H. S. CARSLAW and J. C. JAEGER, *Conduction of Heat in Solids*, Clarendon Press, Oxford (1959).
2. P. M. MORSE and H. FESHBACH, *Methods of Theoretical Physics*, Part II. McGraw-Hill, New York (1961).
3. P. A. LAURA and A. J. FAULSTICH, JR., Unsteady heat conduction in plates of polygonal shape, *Int. J. Heat Mass Transfer* **11**, 297–303 (1968).
4. K. MUNAKATA, On the vibration and elastic stability of a rectangular plate clamped at its four edges, *J. Math. Phys.* **31**, 69–74 (1953).
5. P. A. LAURA, The eigenvalue problem for two-dimensional regions with irregular boundaries, *J. Appl. Mech.* **35** E(1), 198 (March 1968).
6. P. A. LAURA and P. A. SHAHADY, Complex variable theory and elastic stability problems, *J. Engng Mech. Div., ASCE Trans* **95**, 59–67 (1969).
7. P. A. SHAHADY, R. PASAVELLI and P. A. LAURA, An application of complex variable theory to the determination of the fundamental frequency of vibrating plates, *J. Acoust. Soc. Am.* **42**, 806–809 (1967).
8. JAMES, C. M. YU, Application of conformal transformation to variational method: Buckling loads of polygonal plates, presented at the Fifth Southeastern Conference on Theoretical and Applied Mechanics, Raleigh/Durham, 16–17 April, 1970, and accepted for *Developments in Theoretical and Applied Mechanics*, Vol. 5. Pergamon Press, Oxford.
9. P. GLANSDORFF, I. PRIGONGINE and D. F. HAYS, Variational properties of a viscous liquid at a nonuniform temperature, *Physics Fluids* **5**, 144–149 (1962).
10. D. F. HAYS and H. N. CURD, Heat conduction in solids: Temperature-dependent thermal conductivity, *Int. J. Heat Mass Transfer* **11**, 285–295 (1968).
11. D. F. HAYS and H. N. CURD, A variational formulation for diffusion problems: Concentration-dependent diffusivity, *Bull. Acad. Belg. Cl. Sci.* **53**, 469–481 (1967).
12. D. F. HAYS and H. N. CURD, Concentration-dependent diffusion in a semi-infinite medium, *J. Franklin Inst.* **283**, 300–307 (1967).

13. D. F. Hays, An extended variational method applied to Poiseuille flow: Temperature dependent viscosity, *Int. J. Heat Mass Transfer* **9**, 165–170 (1966).
14. D. F. Hays, Variational formulation of the heat equation: Temperature dependent thermal conductivity, *Symposium on Non-Equilibrium Thermodynamics, Variational Techniques and Stability*, pp. 17–43. University of Chicago Press, Chicago (1965).
15. M. Yoshiko and T. Kawai, On the method of application of energy principles to problems of elastic plates, *Proceedings of Eleventh International Congress of Applied Mechanics*, Munich, pp. 461–468 (1964).

APPLICATION DE LA TRANSFORMATION CONFORME ET DE LA MÉTHODE DES VARIATIONS À L'ÉTUDE DE LA CONDUCTION DE CHALEUR DANS DES PLAQUES POLYGONALES AVEC UNE CONDUCTIVITÉ DÉPENDANT DE LA TEMPÉRATURE.

Résumé—Le problème de valeur stationnaire du potentiel local pour la conduction thermique dans une plaque polygonale avec une conductivité dépendant de la température est transformé par une fonction holomorphe en un problème équivalent relatif à une autre plaque ayant une frontière circulaire. Le problème équivalent est alors résolu par la méthode de Raleigh–Ritz puisque les fonctions coordonnées dans la région circulaire sont utilisables. Cette méthode offre une correspondance entre le problème de la distribution de température d'une plaque à toutes les autres. L'application de cette méthode peut être aisément étendue à d'autres phénomènes de transport qui sont gouvernés par l'extrémisation d'une fonctionnelle.

ANWENDUNG DER KONFORMEN ABBILDUNG UND DER VARIATIONSRECHNUNG AUF DAS STUDIUM DER WÄRMELEITUNG IN POLYGONFÖRMIGEN PLATTEN BEI TEMPERATURABHÄNGIGER WÄRMELEITFÄHIGKEIT.

Zusammenfassung—Das stationäre Problem der lokalen Potentialverteilung für die Wärmeleitung in einer polygonförmigen Platte bei temperaturabhängiger Wärmeleitfähigkeit wird durch eine holomorphe Funktion in ein äquivalentes stationäres Problem für eine Platte mit kreisförmiger Berandung transformiert. Das äquivalente Problem wird dann nach der Rayleigh–Ritz-Methode gelöst, da die zugeordneten Funktionen in dem Kreisgebiet einfach zu bestimmen sind. Diese Methode eröffnet einen einheitlichen Zugang zum Problem der Temperaturverteilung in Platten. Die Anwendung dieser Methode lässt sich leicht auf andere Transportphänomene ausdehnen, wenn diese ebenfalls durch eine Extremalbedingung hinsichtlich eines Funktionals beherrscht werden.

ПРИМЕНЕНИЕ МЕТОДА КОНФОРМНОГО ОТОБРАЖЕНИЯ И ВАРИАЦИОННОГО МЕТОДА ДЛЯ ИЗУЧЕНИЯ ТЕПЛОПРОВОДНОСТИ В МНОГОГРАННЫХ ПЛАСТИНАХ ПРИ ЗАВИСИМОСТИ ПРОВОДИМОСТИ ОТ ТЕМПЕРАТУРЫ

Аннотация—Стационарные задачи локального потенциала для теплопроводности в плоском многоугольнике для случая зависящего от температуры коэффициента теплопроводности преобразуются с помощью аналитической функции в эквивалентную плоскую стационарную задачу для кругазатем эквивалентная задача решается методом Релея-Ритца, так как можно выбирать координатные функции для круговой области. Этот метод предлагает унифицированный подход к задаче о распределении температуры в любых плоских областях. Его можно также использовать для изучения других явлений переноса, которые приводятся к определению экстремума некоторого функционала.